CBCS SCHEME

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18MATDIP31

Third Semester B.E. Degree Examination, Dec.2023/Jan.2024
Additional Mathematics – I

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Express $\sqrt{8}+4i$ in the polar form and hence find its modulus and amplitude. (08 Marks)

b. Find the real part of $\dfrac{1}{1+\cos\theta-i\sin\theta}$ (06 Marks)

c. Show that $(1+\cos\theta+i\sin\theta)^n+(1+\cos\theta-i\sin\theta)^n=2^{n+1}\cos^n\left(\dfrac{\theta}{2}\right)\cos\left(\dfrac{n\theta}{2}\right)$ (06 Marks)

OR

2 a. If $\vec{A}=\hat{i}+2\hat{j}+3\hat{k}$, $\vec{B}=-\hat{i}+2\hat{j}+\hat{k}$ and $\vec{C}=3\hat{i}+\hat{j}$, find p such that $\vec{A}+p\vec{B}$ is perpendicular to $\vec{C}$. (08 Marks)

b. Find the area of the parallelogram whose adjacent sides are the vectors $\vec{A}=2\hat{i}+4\hat{j}-5\hat{k}$ and $\vec{B}=\hat{i}+2\hat{j}+3\hat{k}$. (06 Marks)

c. If $\vec{A}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{B}=3\hat{i}-\hat{j}+2\hat{k}$ then show that $\vec{A}+\vec{B}$ and $\vec{A}-\vec{B}$ are orthogonal. (06 Marks)

Module-2

3 a. Obtain the Maclaurin's series expansion of log(sec x) upto the term containing x^3. (08 Marks)

b. Using Euler's theorem, prove that $xu_x+yu_y=\dfrac{5}{2}u$ where $u=\dfrac{x^3+y^3}{\sqrt{x+y}}$ (06 Marks)

c. If $u=f(x-y, y-z, z-x)$, then show that $u_x+u_y+u_z=0$. (06 Marks)

OR

4 a. Prove that

$\sqrt{1+\sin 2x}=1+x-\dfrac{x^2}{2}-\dfrac{x^3}{6}+\dfrac{x^4}{24}+\ldots\ldots\ldots$ by using Maclaurin's series. (08 Marks)

b. If $u=\sin^{-1}\left\{\dfrac{x^2y^2}{x+y}\right\}$, then show that $xu_x+yu_y=3\tan u$, by using Euler's theorem. (06 Marks)

c. If $u=2xy$, $v=x^2-y^2$ and $x=r\cos\theta$, $y=r\sin\theta$, find $\dfrac{\partial(u,v)}{\partial(r,\theta)}$. (06 Marks)

Module-3

5 a. A particle moves along the curve $x=1-t^3$, $y=1+t^2$, $z=2t-5$ where t is time. Find the components of velocity and acceleration at t = 1 in the direction $2\hat{i}+\hat{j}+2\hat{k}$ (08 Marks)

b. Find the unit normal to the surface $xy^3z^2=4$ at $(-1,-1,2)$ (06 Marks)

c. Show that $\vec{F}=(x+y+z)\hat{i}+(x+2y-z)\hat{j}+(x-y+2z)\hat{k}$ is irrotational. (06 Marks)

OR

6 a. Find $\nabla\cdot\vec{F}$ and $\nabla\times\vec{F}$ where $\vec{F}=\nabla(x^3+y^3+z^3-3xyz)$ **(08 Marks)**

b. If $\vec{F}=(x+y+1)\hat{i}+\hat{j}-(x+y)\hat{k}$, then show that $\vec{F}\cdot \text{curl}\,\vec{F}=0$. **(06 Marks)**

c. Find the value of a such that $\vec{F}=(x+3y)\hat{i}+(y-2z)\hat{j}+(x+az)\hat{k}$ is solenoidal. **(06 Marks)**

Module-4

7 a. Evaluate

$$\int_0^{\pi/2}\sin^5 x\,dx$$ **(08 Marks)**

b. Evaluate $\int_0^{\infty}\frac{x^4}{(1+x^2)^4}\,dx$ **(06 Marks)**

c. Evaluate $\iint_R (x^2+y^2)\,dx\,dy$ where R is the region bounded by $y=x$ and $y=x^2$. **(06 Marks)**

OR

8 a. Evaluate

$$\int_0^{\pi/2}\cos^6 x\,dx$$ **(08 Marks)**

b. Evaluate $\int_0^{a} x\sqrt{ax-x^2}\,dx$ **(06 Marks)**

c. Evaluate $\int_0^a\int_0^b\int_0^c (x+y+z)\,dx\,dy\,dz$ **(06 Marks)**

Module-5

9 a. Solve : $y(2x-y+1)dx+x(3x-4y+3)dy=0$ **(08 Marks)**

b. Solve : $\frac{dx}{dy}+\frac{y\cos x+\sin y+y}{\sin x+x\cos y+x}=0$ **(06 Marks)**

c. Solve : $\frac{dx}{dy}+\frac{2y}{x}=y^2x$ **(06 Marks)**

OR

10 a. Solve : $\frac{dy}{dx}+\frac{y}{x}=y^2x$ **(08 Marks)**

b. Solve : $(5x^4+3x^2y^2-2xy^3)dx+(2x^3y)-3x^2y^2-5y^4)dy=0$ **(06 Marks)**

c. Solve : $\frac{dy}{dx}+y\cot x=\cos x$ **(06 Marks)**

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